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On Condensates in Strongly Coupled Gauge Theories

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We present a new perspective on the nature of quark and gluon condensates in quantum chromodynamics. We suggest that the spatial support of QCD condensates is restricted to the interiors of hadrons, since these condensates arise due to the interactions of confined quarks and gluons. An analogy is drawn with order parameters like the Cooper pair condensate and spontaneous magnetization experimentally measured in finite samples in condensed matter physics. Our picture explains the results of recent studies which find no significant signal for the vacuum gluon condensate. We also give a general discussion of condensates in asymptotically free vectorial and chiral gauge theories.

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I. INTRODUCTION

Hadronic condensates play an important role in quantum chromodynamics (QCD). Two important examples are $\langle \bar q q\rangle \equiv \langle \sum_{a=1}^{N_c} \bar q_a q^a\rangle$ and $\langle G_{\mu\nu}G^{\mu\nu}\rangle \equiv \langle \sum_{a=1}^{N_c^2-1} G^a_{\mu\nu}G^{a\,\mu\nu}\rangle$, where q is a light quark (i.e., a quark with current-quark mass small compared with the confinement scale), $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s c_{abc} A^b_\mu A^c_\nu$, a, b, c denote the color indices, and $N_c = 3$. (With our $(+---)$ metric, for a given a, $G^a_{\mu\nu}G^{a\,\mu\nu} = 2(|\mathbf{B}^a|^2 - |\mathbf{E}^a|^2)$.) For QCD with N_f light quarks, the $\langle \bar q q\rangle = \langle \bar q_L q_R + \bar q_R q_L\rangle$ condensate spontaneously breaks the global chiral symmetry $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ down to the diagonal, vectorial subgroup $\mathrm{SU}(N_F)_{diag}$, where $N_f = 2$ (or $N_f = 3$ if one includes the s quark). (Pre-QCD studies of spontaneous chiral symmetry breaking, SχSB, include [1, 2].) In an otherwise massless theory, the $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ condensate breaks dilatation invariance. Conventionally, these condensates are considered to be properties of the QCD vacuum and hence to be constant throughout spacetime [3].

In this paper we will present a new perspective on the nature of QCD condensates $\langle \bar q q\rangle$ and $\langle G_{\mu\nu}G^{\mu\nu}\rangle$, particularly where they have spatial and temporal support. We suggest that their spatial support is restricted to the interiors of hadrons, since these condensates arise due to the interactions of quarks and gluons which are confined within hadrons. Chiral symmetry is thus broken in a limited domain of size $1/m_\pi$. Higher-order condensates such as $\langle (\bar q q)^2\rangle$, $\langle (\bar q q) G_{\mu\nu}G^{\mu\nu}\rangle$, etc. are also present, and our discussion implicitly also applies to these [4].

II. A PICTURE OF QCD CONDENSATES

We first emphasize the subtlety in characterizing the formal quantity $\langle 0|\mathcal{O}|0\rangle$ in a canonical operator-based field theory, where $\mathcal{O}$ is a product of quantum field operators, by recalling that one can render this automatically zero by normal-ordering $\mathcal{O}$. This subtlety is especially delicate in a confining theory, since the vacuum state in such a theory is not defined relative to the fields in the Lagrangian, quarks and gluons, but to the actual physical, color-singlet, states.

The Euclidean path-integral (vacuum-to-vacuum amplitude), Z, provides a precise meaning for the expectation value $\langle \mathcal{O}\rangle = \lim_{J\to 0}(\delta \ln Z/\delta J)$, where J is a source for $\mathcal{O}$. The path integral for QCD, integrated over quark fields and gauge links using the gauge-invariant lattice discretization exhibits a formal analogy with the partition function for a statistical mechanical system. In this correspondence, a condensate such as $\langle \bar q q\rangle$ or $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ is analogous to an ensemble average in statistical mechanics. It is helpful to pursue this analogy in order to develop a physical picture of the QCD condensates. In the context of condensed matter physics, let us consider a phase transition which, for temperature $T < T_c$, produces spontaneous symmetry breaking with an associated nonzero value for some order parameter. For example, in a superconductor, the electron-phonon interaction produces a pairing of two electrons with opposite spins and 3-momenta at the Fermi surface, and, for $T < T_c$, an associated nonzero Cooper pair condensate $\langle ee\rangle_T$ [5], where here $\langle ...\rangle_T$ means thermal average. Since this condensate has a phase, the phenomenological Ginzburg-Landau (GL) free energy function $F_{GL} = |\nabla\Phi|^2 + c_2(\Phi^*\Phi) + c_4(\Phi^*\Phi)^2$ uses a complex scalar field Φ to represent it. The formal treatment of a phase transition in a statistical mechanical system begins with a partition function calculated for a finite d-dimensional lattice, and then takes the thermodynamic (infinite-volume) limit. The non-analytic behavior associated with a phase transition and the nonzero order parameter only occur in this infinite-volume limit. Thus, in the example of the superconductor, for $T < T_c$, the (infinite-volume) system develops a nonzero value of the order parameter, namely $\langle\Phi\rangle_T$, in the phenomenological Ginzburg-Landau model, or $\langle ee\rangle_T$, in the microscopic Bardeen-Cooper-Schrieffer theory. In the formal statistical mechanics context, the minimization of the $|\nabla\Phi|^2$ term implies that the order parameter is a constant throughout the infinite spatial volume.

However, the infinite-volume limit is only an idealization; in reality, superconductivity is experimentally observed to occur in finite samples of material, such as Sn,

Nb, etc., and the condensate clearly has spatial support only in the volume of these samples. This is evident from either of two basic properties of a superconducting substance, namely, (i) zero-resistance flow of electric current, and (ii) the Meissner effect, that $|\mathbf{B}(z)| \sim |\mathbf{B}(0)|e^{-z/\lambda_L}$ for a magnetic field $\mathbf{B}(z)$ a distance z inside the superconducting sample, where the London penetration depth λ_L is given by $\lambda_L^2 = m_e c^2/(4\pi n e^2)$ (n = electron concentration); both of these properties clearly hold only within the sample. The same statement applies to other phase transitions such as liquid-gas or ferromagnetic; again, in the formal statistical mechanics framework, the phase transition and associated symmetry breaking by a nonzero order parameter at low T occur only in the thermodynamic limit, but experimentally, one observes the phase transition to occur effectively in a finite volume of matter, and the order parameter (e.g., Cooper pair condensate or spontaneous magnetization M) has support only in this finite volume, rather than the infinite volume considered in the formal treatment. Similarly, the Goldstone modes that result from the spontaneous breaking of a continuous symmetry (e.g., spin waves in a Heisenberg ferromagnet) are experimentally observed in finite-volume samples. There is, of course, no conflict between the experimental measurements and the abstract theorems; the key point is that these samples are large enough for the infinite-volume limit to be a useful idealization. Standard finite-size scaling methods have long been used in statistical mechanics to relate real finite-size systems and their behavior to the formal infinite-volume limit. In essence, one requires that the minimum dimension of the finite sample must be large compared to the correlation length. Similar descriptions in terms of (effective) phase transition phenomena have been applied recently to numerous nanoscale systems, such as quantum dots [6].

The physics of finite-size condensed-matter systems helps to motivate our analysis for QCD. In our picture, the spatial support for QCD condensates is where the color-nonsinglet particles are whose interactions give rise to them, just as the spatial support of a magnetization M, say, is inside, not outside, of a piece of iron. This conclusion follows from the path integral for QCD; the quark and gluon field configurations that make significant contributions to this integral must be physical configurations, i.e., they must be confined in hadrons. Hence, the same should be true of the quantities involving these fields, in particular, the quark and gluon condensates.

The physical origin of the $\langle\bar{q}q\rangle$ condensate in QCD can be argued to be due to the reversal of helicity (chirality) of a massless quark as it moves outward from the center of a hadron and is reflected back inward at the boundary of a hadron, owing to confinement [7]. This argument implies that the condensate has support only within the spatial extent where the quark is confined; i.e., the physical size of a hadron. Another way to infer this is to note that in the light-front Fock-state picture of hadron wavefunctions [8], a valence quark can flip its chirality when it interacts or interchanges with the sea quarks of multiquark Fock states, thus providing a dynamical origin for the running quark mass. In this description, the $\langle\bar{q}q\rangle$ and $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ condensates are effective quantities which originate from $q\bar{q}$ and gluon contributions to the higher Fock state light-front wavefunctions of the hadron and hence are localized within the hadron.

Let us consider a meson consisting of a light quark q bound to a heavy antiquark, such as a B meson. One can analyze the propagation of the light q in the background field of the heavy $\bar{b}$ quark. Solving the Dyson-Schwinger equation for the light quark one obtains a nonzero dynamical mass and, via the connection mentioned above, hence a nonzero value of the condensate $\langle\bar{q}q\rangle$. But this is not a true vacuum expectation value; instead, it is the matrix element of the operator $\bar{q}q$ in the background field of the $\bar{b}$ quark. The change in the (dynamical) mass of the light quark in this bound state is somewhat reminiscent of the energy shift of an electron in the Lamb shift, in that both are consequences the fermion being in a bound state rather than propagating freely.

Insights into the nature of spontaneous chiral symmetry breaking in QCD can be obtained using approximate solutions of the Dyson-Schwinger equation for a massless quark propagator; if the running coupling $\alpha_s = g_s^2/(4\pi)$ exceeds a value of order 1, this yields a nonzero dynamical (constituent) quark mass Σ [9]. Since in the path integral, Σ is formally a source for the operator $\bar{q}q$, one associates $\Sigma \neq 0$ with a nonzero quark condensate (see also Refs. [10, 11]). However, this application of the Dyson-Schwinger formalism has the defect that it does not incorporate the property of confinement, as is clear from the fact that it is an equation for a quark, but the physical states of the theory are color-singlets, not quarks and gluons. This is a significant defect, since as we have noted, the helicity-reversal of a massless quark as it is reflected inward at the outer boundary of a hadron, due to confinement, provides a physical mechanism for spontaneous chiral symmetry breaking in QCD. Hence, the Dyson-Schwinger equation for the propagator of an isolated quark cannot reliably be used to determine where the quark condensate has spatial support; in particular, it cannot be used to infer that it is a spacetime constant.

Similarly, it is important to use the equations of motion for confined quarks and gluon fields when analyzing current correlators in QCD, not free propagators, as has often been done in traditional analyses of operator products. Since after a $q\bar{q}$ pair is created, the distance between the quark and antiquark cannot get arbitrarily great, one cannot create a quark condensate which has uniform extent throughout the universe.

The Anti-De Sitter/conformal field theory (AdS/CFT) correspondence between string theory in AdS space and CFT's in physical spacetime has been used to obtain an analytic, semi-classical model for strongly-coupled QCD which has scale invariance and dimensional counting at short distances and color confinement at large distances [12]. Color confinement can be imposed by introducing

hard-wall boundary conditions at $z = 1/\Lambda_{QCD}$, where z is the AdS fifth dimension, or by modification of the AdS metric. This AdS/QCD model gives a good representation of the mass spectrum of light-quark mesons and baryons as well as the hadronic wavefunctions [13]. One can also study the propagation of a scalar field $X(z)$ as a model for the dynamical running quark mass [13]. The AdS solution has the form [14] $X(z) = a_1 z + a_2 z^3$, where a_1 is proportional to the current-quark mass. The coefficient a_2 scales as Λ^3_{QCD} and is the analog of $\langle\bar{q}q\rangle$; however, since the quark is a color nonsinglet, the propagation of $X(z)$, and thus the domain of the quark condensate, is limited to the region of color confinement. The AdS/QCD picture of condensates with spatial support restricted to hadrons is in general agreement with results from chiral bag models [15], which modify the original MIT bag by coupling a pion field to the surface of the bag in a chirally invariant manner. Since explicit breaking of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry is small, and hence m_π is small relative to typical hadronic mass scales like m_ρ or m_N, these condensates can be treated as approximately constant throughout much of the volume of a hadron.

Our picture of condensates with spatial support restricted to the interiors of hadrons is consistent with the identification of pions as almost Nambu-Goldstone bosons. In our picture, the pions play a role analogous to the Nambu-Goldstone modes, namely the quantized spin waves (magnons), that are experimentally observed in a piece of a ferromagnetic substance below its Curie temperature. Again, strictly speaking, these spin waves result from the spontaneous breaking of a continuous symmetry, which only occurs in an idealized infinite-volume limit, but this limit provides a very good approximation to a finite-volume sample. The pions are the almost Nambu-Goldstone bosons resulting from the spontaneous breaking of the global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry down to $\mathrm{SU}(2)_{diag.}$, and so, quite logically, the spatial support of their coordinate-space wavefunctions is also a region where the chiral-symmetry breaking quark condensate exists. It is important to recall that the size of a hadron depends not only on confinement but also on the virtual emission and reabsorption of other hadrons, most importantly pions, since they are the lightest. Hence a hadron can be regarded as being surrounded by a cloud of virtual pions. By general quantum mechanical arguments, this cloud is of size $\sim 1/m_\pi$. If the two sources of explicit breaking of chiral $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry were removed, i.e., the m_u and m_d current-quark masses were taken to zero and the electroweak interactions were turned off, so that $m_\pi = 0$, then the size of a hadron, including its pion cloud, would increase without bound (until it impinged on neighboring hadrons). In this case, the quark and gluon condensates would also extend throughout all of spacetime. Thus, our picture of condensates reduces to the conventional view in the chiral limit. This also shows the consistency of our picture with current algebra results such as the Gell-Mann-Oakes-Renner (GMOR) relation, $m_\pi^2 = -f_\pi^{-2}(m_u + m_d)\langle\bar{q}q\rangle$ [16]. Because of confinement, the condensate $\langle\bar{q}q\rangle$ has finite size $\leq 1/m_\pi$. In the chiral limit $\langle\bar{q}q\rangle$ becomes a constant throughout spacetime, consistent with standard analyses. From the GMOR relation, with the current-quark masses $m_u + m_d \simeq 12$ MeV, it follows that $|\langle\bar{q}q\rangle|^{1/3} \simeq 240$ MeV. With the standard convention that these current-quark masses are taken as positive, $\langle\bar{q}q\rangle$ is negative. As one approaches the chiral limit, in our picture, this condensate involves a matrix element of $\bar{q}q$ in the nucleon, whose size is getting very large because of its pion cloud; it thus receives commensurately large contributions from this pion cloud around the nucleon.

We also comment on the nucleon sigma term. Using the state normalization given by $\langle N(\mathbf{p})|N(\mathbf{p}')\rangle = 2p^0\delta^3(\mathbf{p}-\mathbf{p}')$, this term is $\sigma_{\pi N} \equiv \sigma_{\pi N}(0) = (m_u + m_d)(2M_N)^{-1}\langle N(\mathbf{p})|\bar{q}q|N(\mathbf{p})\rangle$. Values of $\sigma_{\pi N}$ extracted from experimental data range from about 45 to 70 MeV [17]. A comparison of this result with the matrix element of $\bar{q}q$ in the GMOR relation would be worthwhile, but, as reviewed by Schweitzer [17], there are many theoretical and model-dependent uncertainties.

Several studies have reported values of the (renormalization-invariant) quantity $\langle(\alpha_s/\pi)G_{\mu\nu}G^{\mu\nu}\rangle$ by analyzing vacuum-to-vacuum current correlators constrained by data for $e^+e^- \to$ charmonium and hadronic τ decays [18]-[20]. In the pioneering work on QCD sum rules [18] the authors obtained an estimate $\simeq 0.01$ GeV^4. Some recent values (in GeV^4) include 0.006 ± 0.012 [20](a), 0.009 ± 0.007 [20](b), and -0.015 ± 0.008 [20](c). These values show significant scatter and even differences in sign. In our analysis the vacuum gluon condensate vanishes; it is confined within hadrons, rather than extending throughout all of space, as would be true of a vacuum condensate.

In our picture, the QCD condensates should be considered as contributing to the masses of the hadrons where they are located. This is clear, since, e.g., a proton subjected to a constant electric field will accelerate and, since the condensates move with it, they comprise part of its mass. Similarly, when a hadron decays to a non-hadronic final state, such as $\pi^0 \to \gamma\gamma$, the condensates in this hadron contribute their energy to the final-state photons. Thus, over long times, the dominant regions of support for these condensates would be within nucleons, since the proton is effectively stable (with lifetime $\tau_p >> \tau_{univ} \simeq 1.4 \times 10^{10}$ yr.), and the neutron can be stable when bound in a nucleus. In a process like $e^+e^- \to$ hadrons, the formation of the condensates occurs on the same time scale as hadronization. In accord with the Heisenberg uncertainty principle, these QCD condensates also affect virtual processes occurring over times $t \lesssim 1/\Lambda_{QCD}$. Our suggestion implies that condensates $\langle\bar{q}q\rangle$ in different hadrons may be chirally rotated with respect to each other, somewhat analogous to disoriented chiral condensates in heavy-ion collisions [21].

Lattice gauge simulations provide a powerful way to investigate properties of QCD, including both hadron

states and quark and gluon condensates [22]. Some early analytic and numerical lattice studies of $\langle\bar{q}q\rangle$ include Refs. [23, 24]. Our suggestion can, in principle, be verified by careful lattice measurements. Note that the lattice measurements that have inferred nonzero values of $\langle\bar{q}q\rangle$ and $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ were performed in finite (Euclidean) volumes, although these were usually considered as approximations to the infinite-volume limit.

III. IMPLICATIONS FOR GENERAL ASYMPTOTICALLY FREE GAUGE THEORIES

Having discussed QCD, we next consider, as an exercise, how our observations apply to other asymptotically free gauge theories. We begin with a vectorial gauge theory with the gauge group $SU(N_c)$, allowing N_c to be generalized to values $N_c \ge 3$. First, consider a theory of this type with no fermions, so that only $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ need be considered. This condensate would then have support within the interior of the glueballs. Second, consider a theory with $N_f = 1$ massless or light fermion transforming according to some nonsinglet representation R of $SU(N_c)$. The $\langle\bar{q}q\rangle$ and $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ condensates in this theory would have support in the interior of the mesons, baryons, and glueballs (or mass eigenstates formed from glueballs and mesons). Here, the condensate $\langle\bar{q}q\rangle$ does not break any non-anomalous global chiral symmetry, so there would not be any Nambu-Goldstone boson (NGB). In both of these theories, the sizes of the mesons, baryons, and glueballs are $\simeq 1/\Lambda$, where Λ is the confinement scale.

We next consider asymptotically free chiral gauge theories (which are free of gauge and global anomalies) with massless fermions transforming as representations $\{R_i\}$ of the gauge group. The properties of strongly coupled theories of this type are not as well understood as those of vectorial gauge theories [25]-[27]. One possibility is that, as the energy scale decreases from large values and the associated running coupling g increases, it eventually becomes large enough to produce a (bilinear) fermion condensate, which thus breaks the initial gauge symmetry [27]. This is expected to form in the most attractive channel (MAC), $R_1 \times R_2 \to R_{cond.}$, which maximizes the quantity $\Delta C_2 = C_2(R_1)+C_2(R_2)-C_2(R_{cond.})$, where $C_2(R)$ is the quadratic Casimir invariant. Depending on the theory, several stages of self-breaking may occur [27, 28]. Let us consider an explicit model of this type, with gauge group SU(5) and massless left-handed fermion content consisting of an antisymmetric rank-2 tensor representation, ψ^{ij}_L, and a conjugate fundamental representation, $\chi_{i,L}$. This theory is asymptotically free and has a formal $U(1)_\psi \times U(1)_\chi$ global chiral symmetry; both U(1)'s are broken by SU(5) instantons, but the linear combination $U(1)'$ generated by $Q = Q_\psi - 3Q_\chi$ is preserved. The MAC for condensation is $10 \times 10 \to \bar{5}$, with $\Delta C_2 = 24/5$, and the associated condensate is $\langle\epsilon_{ijk\ell n}\psi^{jk\ T}_L C\psi^{\ell n}_L\rangle$, which breaks SU(5) to SU(4). Thus, as the energy scale decreases and the running $\alpha = g^2/(4\pi)$ grows, at a scale Λ at which $\alpha\Delta C_2 \sim O(1)$, this condensate is expected to form. Without loss of generality, we take $i = 1$, and note

$$\langle\epsilon_{1jk\ell n}\psi^{jk\ T}_L C\psi^{\ell n}_L\rangle \ \propto\ \langle\psi^{23\ T}_L C\psi^{45}_L - \psi^{24\ T}_L C\psi^{35}_L + \psi^{25\ T}_L C\psi^{34}_L\rangle \qquad (1)$$

The nine gauge bosons in the coset SU(5)/SU(4) gain masses of order Λ. The six components of ψ^{ij}_L involved in the condensate (1) also gain dynamical masses of order Λ. These components bind to form an SU(4)-singlet meson whose wavefunction is given by the operator in (1). This binding involves the exchange of the various (perturbatively massless) gauge bosons of SU(4). The condensate (1) breaks the global $U(1)'$, but the would-be resultant NGB is absorbed by the gauge boson corresponding to the diagonal generator in SU(5)/SU(4). We infer that this condensate (1) has spatial support in the meson with the same wavefunction. Aside from the SU(4)-singlet $\chi_{1,L}$, the remaining massless fermion content of the SU(4) theory is vectorial, consisting of a 4, ψ^{1j}_L, and a $\bar{4}$, $\chi_{j,L}$, $j = 2...4$. The formal global flavor symmetry of this effective SU(4) theory at energy scales below Λ is $U(1)_L \times U(1)_R = U(1)_V \times U(1)_A$, and the $U(1)_A$ is broken by SU(4) instantons. This low-energy effective field theory is asymptotically free, so that at lower energy scales, the coupling α that it inherits from the SU(5) theory continues to increase, and the theory confines and produces the condensate $\langle\psi^{1j\ T}_L C\chi_{j,L}\rangle$, which preserves the gauged SU(4) and global $U(1)_V$. We infer that $\langle\psi^{1j\ T}_L C\chi_{j,L}\rangle$ and the SU(4) gluon condensate $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ have spatial support in the SU(4)-singlet baryon, meson, and glueball states of this theory.

Although our suggestion associates condensates in a confining gauge theory G with G-singlet hadrons, these condensates can affect properties of G-singlet particles if they both couple to a common set of fields. For example, the $\langle\bar{F}F\rangle$ condensate and the corresponding dynamical mass Σ_F of technifermions in a technicolor (TC) theory give rise to the masses of the (TC-singlet) quarks and leptons via diagrams involving exchanges of virtual extended technicolor gauge bosons.

IV. STRONGLY COUPLED QED

Our argument is only intended to apply to asymptotically free gauge theories. However, we offer some remarks on the situation for a particular infrared-free theory here, namely quantum electrodynamics (QED), based on a U(1) gauge group with gauge coupling e and some set of fermions ψ_i with charges q_i. Here there are several important differences with respect to an asymptotically free non-abelian gauge theory. First, while the chiral limit of QCD, i.e., quarks with zero current-quark masses, is well-defined because of quark confinement, a

U(1) theory with massless charged particles is unstable, owing to the well-known fact that these would give rise to a divergent Bethe-Heitler pair production cross section. It is therefore necessary to break the chiral symmetry explicitly with bare fermion mass terms m_i. If the running coupling $\alpha_1 = e^2/(4\pi)$ at a given energy scale μ were sufficiently large, $\alpha_1(\mu) \gtrsim O(1)$, an approximate solution to the Dyson-Schwinger equation for the propagator of a fermion ψ_i with $m_i << \mu$ would suggest that this fermion gains a nonzero dynamical mass Σ_i [9, 29] and hence, presumably, there would be an associated condensate $\langle\bar{\psi}_i\psi_i\rangle$ (no sum on i). However, in analyzing SχSB, it is important to minimize the effects of explicit chiral symmetry breaking due to the bare masses m_i. The infrared-free nature of this theory means that for any given value of α_1 at a scale μ, as one decreases m_i/μ to reduce explicit breaking of chiral symmetry, $\alpha_1(m_i)$ also decreases, approaching zero as $m_i/\mu \to 0$. Since $\alpha_1(m_i)$ should be the relevant coupling to use in the Dyson-Schwinger equation, it may in fact be impossible to realize a situation in this theory in which one has small explicit breaking of chiral symmetry and a large enough value of $\alpha_1(m_i)$ to induce spontaneous chiral symmetry breaking. A full analysis would require knowledge of the bound state spectrum of the hypothetical strongly coupled U(1) theory, but this spectrum is not reliably known.

V. OTHER FIELD THEORIES

One could also consider supersymmetric SU(N_c) gauge with a sufficiently small number N_f of light chiral superfields that the theory is in the phase with confinement and spontaneous chiral symmetry breaking. This theory produces both a gluino and a quark condensate. With the supersymmetry unbroken, the physical states are SU(N_c)-singlet hadrons with degenerate superpartners. The extension of our observation for QCD to this theory would lead one to infer that the various condensates reside within these color singlet states.

We also comment on gauge theories in $d = 2$ spacetime dimensions. These include the Schwinger model, QED$_2$ with a massless charged fermion [30] and generalizations thereof to QED$_2$ with a set of N_i copies of massless fermions of different charges q_i, and the 't Hooft model [31], namely the limit $N_c \to \infty$ limit, with g^2N_c fixed, of the U$(N_c)_2$ gauge theory with one or more massive or massless fermions. These theories have the appeal that they are exactly solvable. There are well-known differences between them and real QCD, since in $d = 2$ dimensions (i) there are no dynamical gauge degrees of freedom, (ii) they are super-renormalizable, and (iii) the Mermin-Wagner-Coleman (MWC) theorem [32] forbids the breaking of a continuous global symmetry. Nevertheless, the property that the potential energy associated with a fermion-antifermion pair separated by a distance x increases linearly with x in a U$(1)_2$ or U$(N_c)_2$ gauge theory, and the related absence of any gauge-nonsinglet states in the spectrum, are reminiscent of confinement in QCD. Thus, both of these theories were used as early models exhibiting free behavior at short distances together with the absence of gauge-nonsinglet physical states, as in QCD [31, 33].

The Schwinger model has a spectrum consisting of a free scalar with mass given by $m^2 = e^2/\pi$ and (assuming a particular normal-ordering prescription) exhibits a nonzero $\langle\bar{\psi}\psi\rangle \propto m$, and this is a constant in spacetime. This does not contradict our observation about QCD, however, because the existence of this $\langle\bar{\psi}\psi\rangle \neq 0$ depends crucially on the fact that the U$(1)_A$ chiral symmetry is anomalous;

$$\partial_\mu J^{\mu 5} = \frac{e^2}{2\pi}\,\epsilon_{\mu\nu} F^{\mu\nu} \tag{2}$$

(since otherwise $\langle\bar{\psi}\psi\rangle$ is forced to vanish by the MWC theorem). As is evident from eq. (2), the necessary condition for the U$(1)_A$ symmetry to be anomalous is that $F^{01} = E \neq 0$. Recalling that there are no dynamical gauge fields in $d = 2$, one sees that E is a constant, external electric field. Hence, in this model $\langle\bar{\psi}\psi\rangle$ arises not as a consequence of any dynamical gauge interactions, but instead as a consequence of the imposition of a nonzero external electric field, analogous to the Stark effect. Moreover, the generalization with N_i copies of massless fermions ψ_i with different charges q_i is also exactly solvable, and, in agreement with the MWC theorem, $\langle\bar{\psi}_i\psi_i\rangle = 0$ (no sum on i) if $N_i \geq 2$ (e.g., [35]).

Similar comments apply for the 't Hooft model (with massless fermions); again, a nonzero condensate is only allowed in the case of one flavor, and it owes its existence to the fact that the abelian U(1) factor in the U(N_c) = SU(N_c) × U(1) gauge group gives rise to an equation analogous to eq. (2), so that, the anomaly in the axial-vector current is due to a nonzero value of an external chromoelectric field rather than to any intrinsic dynamics of the theory.

The existence of condensates which are spacetime constants in models without confinement, such as the (4D) NJL model and the ($N \to \infty$ limit of the) 2D model with a four-fermion interaction [36] are fully in agreement with our observation, since neither of these models exhibits confinement.

VI. QCD AT FINITE TEMPERATURE

So far, we have discussed QCD and other theories at zero temperature. For QCD in thermal equilibrium at a finite temperature T, as T increases above the deconfinement temperature T_{dec}, both the hadrons and the associated condensates eventually disappear, although experiments at CERN and BNL-RHIC show that the situation for $T \gtrsim T_{dec}$ is more complicated than a weakly coupled quark-gluon plasma. Our picture of the QCD condensates here is especially close to experiment, since, although finite-temperature QCD makes use of the formal

thermodynamic, infinite-volume limit, actual heavy ion experiments and the resultant (effective) phase transition from confined to deconfined quarks and gluons takes place in the finite volume and time interval provided by colliding heavy ions.

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[1] M. Gell-Mann and M. Lévy, Nuovo Cim. **16** (1960) 705.

[2] Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[3] Here and below, unless otherwise stated, we consider theories at temperature $T = 0$ and chemical potential $\mu = 0$; in particular, we consider times long after the QCD deconfinement-confinement phase transition that occurred at $t \sim 10^{-5}$ sec in the early universe as T decreased below $T_{dec} \simeq 200$ MeV.

[4] The fact that QCD and resultant strong interactions experimentally conserve P and T shows that P- and T-non-invariant condensates such as $\langle G_{\mu\nu}\tilde{G}^{\mu\nu}\rangle$, where $\tilde{G}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$, are negligible. (Explaining this is part of the strong CP problem.)

[5] See, e.g., A. Fetter and J. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971); M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996);

[6] See, e.g., C. Yannouleas and U. Landman, Phys. Rev. Lett. **82** (1999) 5325; Rep. Prog. Phys. **70** (2007) 2067; W. Hofstetter and H. Schoeller, Phys. Rev. Lett. **88** (2002) 106803; M. R. Galpin, D. E. Logan, and H. R. Krishnamurthy, Phys. Rev. Lett. **94** (2005) 186406; G. Zarand, C.H. Chung, P. Simon, and M. Vojta, Phys. Rev. Lett. **97** (2006) 166802.

[7] A. Casher, Phys. Lett. B **83** (1979), 395;

[8] S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Phys. Rept. **301**, 299 (1998).

[9] Early papers include K. Lane, Phys. Rev. D **10** (1974) 2605; H. D. Politzer, Nucl. Phys. B **117** (1976), 397; T. Kugo, Phys. Lett. B **76** (1978) 625. H. Pagels, Phys. Rev. D **19** (1979) 3080; K. Higashijima,, Phys. Rev. D **29** (1984) 1228.

[10] J. M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D **10**, 2428 (1974);

[11] S. Coleman, E. Witten, Phys. Rev. Lett. **45**, 100 (1980).

[12] For a review and references, see S. J. Brodsky and G. F. de Téramond, arXiv:0802.0514 [hep-ph].

[13] S. J. Brodsky and G. F. de Téramond, Phys. Lett. B **582**, 211 (2004); G. F. de Téramond and S. J. Brodsky, Phys. Rev. Lett. **94**, 201601 (2005); S. J. Brodsky and G. F. de Téramond, Phys. Rev. Lett. **96**, 201601 (2006) and to appear.

[14] J. Erlich, E. Katz, D. T. Son, and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005); L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005).

[15] A. Chodos, C. B. Thorn, Phys. Rev. D **12**, 2733 (1975); G. E. Brown, M. Rho, Phys. Lett. B **82**, 177 (1979); A. Hosaka H. Toki, Phys. Rept. **277**, 65 (1996).

[16] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175** (1968) 2195.

[17] See, e.g., P. Schweitzer, Phys. Rev. D **69**, 034003 (2004), and references therein to the extensive literature on $\sigma_{\pi N}$.

[18] M. A. Shifman, A. I. Vainshtein, and V. Zakharov, Nucl. Phys. B **147**, 384, 448 (1979).

[19] S. Narison, *QCD Spectral Sum Rules* (World Scientific, Singapore, 1989).

[20] (a) R. Barate et al., Phys. Rept. **294**, 1 (1998); (b) B. V. Geshkenbein, B. L. Ioffe, K. N. Zyablyuk, Phys. Rev. D **64**, 093009 (2001); B. L. Ioffe, K. N. Zyablyuk, Eur. Phys. J. C **27**, 229 (2003); (c) M. Davier, A. Höcker, Z. Zhang, hep-ph/0701170; M. Davier et al., arXiv:0803.0979.

[21] J. D. Bjorken, SLAC-PUB-5673 (1991); and in *Workshop on Continuous Advances in QCD*, 1994, SLAC-PUB-6488; a review is B. Mohanty and J. Serreau, Phys. Rept. **414**, 263 (2005).

[22] Early reviews include M. Creutz, *Quarks, Gluons, and Lattices* (Cambridge University Press, Cambridge, 1983); J. Kogut, Rev. Mod. Phys. **55**, 775 (1983), and I. Montvay and G. Münster, *Quantum Fields on a Lattice* (Cambridge University Press, Cambridge, 1994). Recent work is reported in the annual conferences, e.g. Lattice 2006 (Tucson) and Lattice 2007 (Regensburg).

[23] J. M. Blairon, R. Brout, F. Englert, and J. Greensite, Nucl. Phys. B **180** (1981) 439; H. Kluberg-Stern, A. Morel, O. Napoly, and B. Petersson, Nucl. Phys. B **190** (1981) 504.

[24] H. Hamber and G. Parisi, Phys. Rev. Lett. **47** (1981) 1792; E. Marinari, G. Parisi, and C. Rebbi, Phys. Rev. Lett. **47** (1981) 1795; J. B. Kogut, M. Stone, H. W. Wyld, J. Shigemitsu, S. H. Shenker, and D. K. Sinclair, Phys. Rev. Lett. **48** (1982) 1140; Nucl. Phys. B **225** (1983) 326; J. B. Kogut, J. Shigemitsu, and D. K. Sinclair, Phys. Lett. B **138** (1984) 283.

[25] G. 't Hooft, in *Recent Developments in Gauge Theories* (1979 Cargèse Institute) (Plenum, New York, 1980).

[26] S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. B **173**, 208 (1980).

[27] S. Raby, S. Dimopoulos, and L. Susskind, Nucl. Phys. B **169**, 373 (1980).

[28] T. Appelquist and J. Terning, Phys. Rev. D **50**, 2116 (1994); T. Appelquist and R. Shrock, Phys. Lett. B **548**, 204 (2002); Phys. Rev. Lett. **90**, 201801 (2003)..

[29] K. Johnson, M. Baker, and R. Willey, Phys. Rev. **136** (1964) B1111; M. Baker and K. Johnson, Phys. Rev. D **3**, 2516 (1971).

[30] J. Schwinger, Phys. Rev. **128** (1962) 2425.

[31] G. 't Hooft, Nucl. Phys. B **72** (1974) 461; *ibid.* **75** (1974) 461.

[32] N. D. Mermin and H. Wagner, Phys. Rev. Lett. **17** (1966) 1133; S. Coleman, Commun. Math. Phys. **31** (1973) 259.

[33] A. Casher, J. B. Kogut, and L. Susskind, Phys. Rev. D **10** (1974) 732.

[34] T. Eller, H. C. Pauli, and S. J. Brodsky, Phys. Rev. D **35** (1987) 1493.

[35] See, e.g., A. Abada and R. E. Shrock, Phys. Lett. B **267** (1991) 282.

[36] D. J. Gross and A. Neveu, Phys. Rev. D **10** (1974) 3235.